

## SIX SIGMA MECHANICAL DESIGN TOLERANCING

by

#### Mikel J. Harry

Principal Staff Engineer Advanced Quantitative Research Laboratory Communications Division Government Electronics Group, Motorola Inc.

#### Reigle Stewart

Staff Engineering Specialist Group Mechanical Design Technology Government Electronics Group, Motorola Inc.

#### **ABSTRACT**

This booklet presents the rationale and methodology for establishing, analyzing, optimizing, and allocating mechanical assembly tolerances. The presentation progresses step by step from the underlying statistical methods to "real world" mechanical design applications. Both the discussion and examples of design analysis and optimization are constructed around the philosophy of Motorola's Six Sigma Program – the creation of product designs that are robust to natural sources of manufacturing and material variation.

The booklet opens with a brief historical account of how human need has motivated the evolution of mechanical tolerances. Certain key events that played a major role in this evolution are highlighted. This discussion closes with an explanation of why it is absolutely essential for the technical community of Motorola Inc. to continue its efforts with respect to "designing for manufacturability."

Following the background information, the statistical principles underlying design analysis are presented and thoroughly discussed from a theoretical as well as a pragmatic frame of reference. Particularly strong emphasis is given to the arithmetic mean  $(\mu)$ , standard deviation  $(\sigma)$ , standard transform (Z), and normal distribution. The discussion then turns to how basic statistical tools are used to form various indices of manufacturing process capability  $(C_p$  and  $C_{pk})$ . In each instance where an analytical tool is presented, it is shown how the supporting statistical principles and equations are shaped and subsequently applied to a generic product referred to as a "widget." This particular product example is interwoven throughout the entire booklet to create a common denominator. In addition, two case studies are presented. The first case involves design optimization of a fairly simple assembly chassis. The second study presents the outcomes of a design optimization related to a sophisticated hand-held military radio.

Directly following the discussion on basic statistics, worst-case analysis is introduced. Although worst-case analysis is not considered a statistical procedure, it has been included for convenience; that is, a means for benchmarking the relative efficacity of comparable statistical procedures. The first statistical tolerance analysis procedure discussed is the "Root-Sum-of-Squares" method, or simply RSS. Focus is on the major reason for using RSS analysis: taking advantage of the probabilities related to tolerance stacking. Also discussed are the limitations of RSS analysis. As an alternative to RSS analysis, a method called "Dynamic Root-Sum-of-Squares," or DRSS, is introduced. In addition, a second method called "Static Root-Sum-of-Squares" (SRSS) also is presented. Both methods take into account shifts and drifts in the process mean which normally result from such factors as tool wear and machine set-up. With DRSS, as well as SRSS, the design engineer structures a baseline design and then computes the probability of assembly. If the outcome is not relatively close to Motorola's Six Sigma criteria (99.99966 percent likelihood of assembly, in either direction of the nominal assembly gap) the engineer conducts an optimization and allocation study using the statistically-based equations presented in this booklet. The end result is a mechanical assembly that exhibits 60 performance at the assembly level of analysis.

The booklet closes with a discussion of a statistical procedure that compensates for short-run production. Implications and limitations of this particular practice are discussed from a pragmatic point of view. In addition, other areas of application within the field of mechanical engineering are discussed. As a final point, the role of computer hardware and software for use in mechanical design analysis and optimization is highlighted.



MOTOROLA INC.

# Ordering Instructions for "Six Sigma Mechanical Design Tolerancing"

Publication Number 6σ-2-10/88

Send a memo to: Motorola Government Electronics Group 8201 E. McDowell Rd. Scottsdale, AZ 85257

Attn: Public Relations, Rm H3184 Phone (602) 990-5716

Include in the request: • Publication number

- · Requestor's name
- Department number to be charged
- Requestor's address and telephone number
- · Quantity requested

© Copyright 1988 by Motorola Inc. Printed in the U.S.A.

 $6\sigma\text{-}2\text{-}10/88$ 

#### TABLE OF CONTENTS

SECTIO		ge
Introdu		1
1.1 1.2	The Evolution of Mechanical Tolerances The Concept of Designing for Producibility	1
1.2	The Concept of Designing for Froductority	l
SECTIO		
Statistic	al Principles Underlying Design Analysis	3
2.1	Measures of Central Tendency and Variability	3
2.2	Graphical Display of Variation	4
2.3		5
2.4		7
2.5 2.6	• •	8
2.7	- 마이트	1
2.7	Definition of the Capability	1
SECTIO		
Statistic		3
3.1		3
3.2		6
3.3		20
3.4	Static Root-Sum-of-Squares (SRSS) Analysis	23
SECTIO	N 4	
		26
4.1	Optimizing the Nominal Dimensions of the Components	26
4.2	Optimizing the Tolerances of the Components	27
4.3	Establishing the Variance Pool	27
SECTIO	N 5	
		30
Design .	maryons and optimization duractimes	U
SECTIO		
Case Sti		34
6.1		34
6.2	AN/PRC-112(V) Radio Set Case Study	0
SECTIO	N 7	
		9
7.1		9
7.2	Considerations for Short-Run Production	19
7.3		2
7.4		2
7.5	Further Direction for Design Analysis and Optimization	2
<b>ABOUT</b>	THE AUTHORS	3
<b>ACKNO</b>	WLEDGEMENTS5	4
REFERI	ENCES 5	4
		5
		7
	and the control of th	1
APPENI	DIX B – Mechanical Design Analysis and Optimization Spreadsheet for AN/PRC-112(V) Radio Set Case Study	O

### LIST OF ILLUSTRATIONS

Number		Page
1-1	Example of a Dimensional Tolerance	1
1-1 2-1	Till Widget Droduct	3 5
2-1 2-2		6
2-3	Turnical Areas Under the Normal Curve	U
2-4		7
	At 1 Distribution	9
2-5	Concepts Underlying $C_p$ and $C_{pk}$	10
2-6	Centered $\pm 6\sigma$ Distribution where $C_p = 2.0$ and $C_{pk} = 2.0$	11
2-7	Entered $\pm 6\sigma$ Distribution where $C_p = 2.0$ and $C_{pk} = 2.0$ . $C_{pk} = 1.5$ . $\pm 6\sigma$ Distribution with a 1.5 $\sigma$ Mean Offset where $C_p = 2.0$ , $C_{pk} = 1.5$	15
3-1	Widest Loop Applyers	
3-2	Monte Carlo Simulation of the Widget Assembly Gap Under the Constraint	18
	of $C_p = 1.0$ , $C_{pk} = 1.0$ for Each Component  The Effect of Drift as a Function of Tool Wear Over Time	20
3-3	The Effect of Drift as a Function of Tool wear Over Time	
3-4	Monte Carlo Simulation of the Widget Assembly Gap Based on a 1.5 $\sigma$	22
	Cumulative Change in the Mean of Each Component	34
6-1	Rendition of the Chassis Assembly	35
6-2	Baseline Design of the Chassis Assembly	41
6-3	Photos of the AN/PRC-112(V) Radio Set	42
6-4	Selected Engineering Drawings of the AN/PRC-112(V) Radio Set  Selected Engineering Drawings of the AN/PRC-112(V) Radio Set	43
6-5	Illustration of Module U5 in the AN/PRC-112(V) Radio Set	45
6-6	Baseline Analysis Inputs for AN/PRC-112(V) Radio Set	46
6-7	Summary Spreadsheet for AN/PRC-112(V) Design Optimization Study  Summary Spreadsheet for AN/PRC-112(V) Design Optimization Study	
6-8	Illustration of Tolerance Allocation for a Typical Module in the AN/PRC-112(V) Radio Set	48
	Radio Set	
7-1	Impact of Product Complexity on the Widget Assembly Gap for	50
	Several Levels of Manufacturing Capability  Several Levels of Manufacturing Capability	
7-2	Impact of Product Complexity on the Widget Assembly Gap for Several Mean Shift Conditions Under the Constraint $C_p = 1.0$	50
	Several Mean Shift Conditions Under the Constraint Cp = 1.0	
7-3	Impact of Product Complexity on the Widget Assembly Gap for Several Mean Shift Conditions Under the Constraint $C_p = 2.0$	51
	Several Mean Shift Conditions Under the Constraint Cp = 2.0  Several t Distributions Compared to the Normal Distribution	51
7-4	Several t Distributions Compared to the Normal Distribution	
* *0	ST OF TABLES	
LIS		W 60
6-1	Baseline Design Analysis Parameters Related to the Chassis Assembly Case Study	36
6-2	p.gg. 4 . L Desulte Deleted to the Chassis Assembly Case Study	36
6-3	DRSS Analysis Results Related to the Chassis Assembly Case Study	37
	DDCC Design Optimization Parameters Associated Will life	
6-4	Classic Assembly Case Study	38
6-5	DDSS Design Optimization Statistics Related to the Chassis Assembly	20
0-3	Name of Dimensions	38
6-6	DRSS Design Optimization Outcomes Related to the Chassis Assembly	.00
0-0	<u></u> .	39
6-7	Political to the AN/PRC-112(V) Radio Set	44
6-8	G assign of Design Analysis Results for the AN/PRC-112(V) Radio Sci	47
6-9	Common of the Design Optimization Study Related to the Alvir RC-112(1)	
0 /	Radio Set	47